

Partial $N = 2 \rightarrow N = 1$ Local Supersymmetry Breaking and Solvable Lie Algebras

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Generic partial supersymmetry breaking of $N = 2$ supergravity with zero vacuum energy and with surviving unbroken arbitrary gauge groups is exhibited. Specific examples are given.

1. Introduction

A great number of considerable results towards a deeper understanding of string-string dualities [1–7] together with non-perturbative aspects of field and string theory [8–11], has been recently achieved within the framework of $N = 2$ supersymmetry. Among the reasons for so much interest devoted in the last decade to $N = 2$ supersymmetric theories is the rich geometrical structure of their scalar manifold $\mathcal{M}_{\text{scalar}}$, which has the form:

$$\mathcal{M}_{\text{scalar}} = \mathcal{SK}_n \otimes \mathcal{Q}_m \quad (1)$$

where \mathcal{SK}_n denotes a complex n -dimensional special Kähler manifold [12–16] (for a review of Special Kähler geometry see either [17] or [18]) and \mathcal{Q}_m a m -dimensional quaternionic manifold, n being the number of vector multiplets and m the number of hypermultiplets [19–21,15].

Unfortunately no phenomenological prediction can be drawn at an $N = 2$ level, mainly because of the presence of *mirror fermions* which cause these theories to be non-chiral. Within the supersymmetry framework, only $N = 1$ theories meet the

phenomenological requirement for a chiral theory. In order to extend the results obtained in $N = 2$ theories to an $N = 1$ level, much interest has been recently devoted towards finding a mechanism of spontaneous $N = 2 \rightarrow N = 1$ supersymmetry breaking. On the other hand, there are hints that such a spontaneous SUSY breaking should occur from a non-perturbative analysis of $N = 2$ supergravities based on the study of extremal black hole solutions [37].

Results obtained time ago, in the context of superconformal tensor calculus lead to a “no-go” theorem, stating the impossibility for a spontaneous $N = 2$ to $N = 1$ SUSY breaking to occur [22]. Eventually, with the developments in special Kähler geometry [4] [5][23], this theorem was understood to be a consequence of unnecessary restrictions imposed on the formulation of special Kähler geometry [24–26,28] (namely the condition of the existence of a holomorphic *prepotential* function) and could be removed within an extended definition of special Kähler manifolds. As a matter of fact, the more general models without a prepotential can be formally obtained by combining superconformal tensor calculus (SCT) with appropriate symplectic trans-

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formations which bring SCT models into non-physically equivalent ones.

The first achievements in finding a mechanism of partial supersymmetry breaking have been recently obtained both in a global $N = 2$ supersymmetric minimal model [25] and in a local one [24]. The former model can be obtained from the latter by means of a suitable flat limit. In the local minimal model [24], in which supergravity was coupled with one vector multiplet and one hypermultiplet, a symplectic gauge was used which is compatible with the T-duality of string theory, and is allowed only in the extended formulation of special Kähler geometry.

In a recent publication [29] we succeeded in extending the results obtained in [24] to an $N = 2$ supergravity theory coupled to an arbitrary number n of vector multiplets and m of hypermultiplets, breaking spontaneously to an $N = 1$ theory with the survival of an unbroken arbitrary compact gauge group (for a general review about spontaneous supersymmetry breaking see also [27]). The formalism adopted for describing a generic $N = 2$ matter-coupled theory is consistent with the conventions of [30].

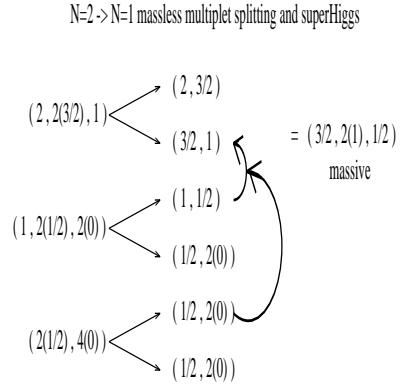
A specific example was worked out [29], corresponding to the choice $\mathcal{SK}_n = SU(1,1)/U(1) \otimes SO(2,n)/SO(2) \times SO(n)$ for the special Kähler manifold and $\mathcal{Q}_m = SO(4,m)/SO(4) \times SO(m)$ for the quaternionic one.

2. Partial $N = 2$ supersymmetry breaking: general features and results

The partial $N = 2$ supersymmetry breaking in a supergravity model is a consequence of a *super-Higgs* mechanism and requires the following minimal ingredients in order to take place:

- the gauging of a gauge group \mathbb{R}^{*2} by the graviphoton A_μ^0 and a vector field A_μ^1
 - one hypermultiplet (the *hidden sector*) charged with respect to the \mathbb{R}^{*2} group

$N = 2$ supersymmetry is violated by spontaneously breaking the $O(2)$ symmetry interchanging the two gravitinos. One of the latters acquires mass by “eating” a fermion field belonging-



ing to the vector multiplet of A_μ^1 , and consequently becoming the top spin state of an $N = 1$ massive spin-3/2 multiplet. The latter, in order to be completed, needs two massive spin-1 fields which are provided by the graviphoton A_μ^0 and A_μ^1 . These two vector fields acquire mass by “eating” two real scalar fields from the hypermultiplet, by means of an ordinary *Higgs* mechanism. The remaining spin-1/2 Majorana field in the vector multiplet fits the lowest spin state of the massive spin-3/2 $N = 1$ multiplet. Therefore, as a consequence of this mechanism, we are left with a massless $N = 1$ graviton multiplet, two massless *chiral* $N = 1$ multiplet and a massive spin-3/2 $N = 1$ multiplet.

The mathematical formulation of the problem is based on the following general property: in an N -extended supergravity theory describing a certain number of scalar fields Φ^I , a *bosonic background* (Φ_o^I) admitting r *killing spinors* is not only an extremum (vacuum) of the scalar potential but it also breaks $N - r$ of the initial N supersymmetries. By *bosonic background* we mean a state defined by a Minkowskian metric, vanishing expectation value of the vector fields and a constant scalar field configuration $\Phi^I = \Phi_o^I$. A *killing spinor* is a constant spinorial parameter such that the corresponding supersymmetry transformation of the spinor fields vanishes. Since the supersymmetry variation of the gravitino on a bosonic background is proportional to the contraction of the gravitino mass-matrix with the SUSY parameter, it is necessary for the gravitino

mass-matrix to have a null eigenvalue in order for a killing spinor to exist.

The occurrence of a partial $N = 2 \rightarrow N = 1$ supersymmetry breaking depends on the choice of the symplectic gauge for the vector fields, since different symplectic gauges may lead, after gauging, to inequivalent theories [17][30].

As previously pointed out, we solved explicitly the case of an $N = 2$ supergravity in which the scalars span the manifolds: $\mathcal{SK}_n = SU(1,1)/U(1) \otimes SO(2,n)/SO(2) \times SO(n)$; $\mathcal{Q}_m = SO(4,m)/SO(4) \times SO(m)$ [29]. Within the coordinate-free definition of special Kähler manifold [14], [15], we chose the symplectic gauge corresponding to the Calabi–Visentini coordinates (S, y^α) [17][23] for \mathcal{SK}_n , which is compatible with the T-duality symmetry in string theory. Moreover we gauged a group of the form:

$$G_{gauge} = \mathbb{R}^{*2} \otimes G_{compact} \quad (2)$$

where the non-compact factor \mathbb{R}^{*2} , responsible for the partial supersymmetry breaking in the way described above, is a subgroup of the isometry group \mathcal{Q}_m , while the compact factor which will survive as the gauge group at the $N = 1$ level, consists of isometries of either \mathcal{SK}_n or \mathcal{Q}_m .

Denoting by q^a , ($a = 1, \dots, 4m$) the coordinates on \mathcal{Q}_m , for a bosonic background defined by a generic point (S, q^a) on the $y^\alpha = 0$ hypersurface of \mathcal{M}_{scalar} , suitable generators of G_{gauge} have been found within the isometry algebra of $\mathcal{M}_{scalars}$, such that a killing spinor exists. The generators T_0, T_1 of the \mathbb{R}^{*2} factor, which serve the purpose, have, in the *canonical* basis of $\mathbf{so}(4,\mathbf{m})$, the form:

$$T_\Lambda(q) = \mathbb{L}(q)T_\Lambda\mathbb{L}(q)^{-1} \quad \Lambda = 0, 1 \quad (3)$$

$$T_0 = E_{\epsilon_{l-3}-\epsilon_{l-1}} + E_{\epsilon_{l-3}+\epsilon_{l-1}} \quad (4)$$

$$T_1 = -(E_{\epsilon_{l-3}+\epsilon_l} + E_{\epsilon_{l-3}-\epsilon_l}) \quad (5)$$

where we have denoted by $\mathbb{L}(q)$ the coset representative on \mathcal{Q}_m . For this result to be true, the coupling constants g and g' , associated with the \mathbb{R}^{*2} generators, are required to fulfill the condition $g = g'$, which ensures the existence of a vanishing eigenvalue for the gravitino mass matrix, the corresponding eigenvector being the killing

spinor. The mass of the massive gravitino is found to be proportional to $g + g' = 2g$. Therefore the choice of the Calabi–Visentini parametrization for the special Kähler manifold and a suitable choice of gauging allowed to define a theory in which partial SUSY breaking $N = 2 \rightarrow N = 1$ occurs. Moreover this mechanism leaves the gauge symmetry corresponding to a $G_{compact} = SO(n - 1)$ unbroken at the $N = 1$ level.

A deeper understanding of the physical meaning underlying the choice made for the generators of G_{gauge} , may be achieved using Alekseevskii's representation of quaternionic manifolds [31] [36](see also [32–34]). This mathematical formulation allows to describe \mathcal{Q}_m as a group manifold generated by a *solvable* Lie algebra V_m so that the hyperscalars are just coordinates parametrizing it. The generators chosen for the group \mathbb{R}^{*2} belong to a *maximal abelian ideal* $\mathcal{A} \subset V_m$ and are parametrized by two hyperscalars (t^0, t^1) on which the non-compact abelian group act as traslations. It turns out that these two fields define flat directions of the scalar potential.

In the light of the results obtained in a recent paper by some of us [40], the nilpotent elements of the maximal abelian algebra \mathcal{A} are naturally related to the Peccei–Quinn symmetry generators and the two scalars t^0, t^1 may be interpreted as Ramond–Ramond fields. The partial SUSY breaking $N = 2 \rightarrow N = 1$ may thus be thought of as an effect of the “condensation” of the two R–R fields t^0, t^1 in the vacuum state [38][39].

In conclusion, from our analysis it follows that $N = 2$ supergravity can be spontaneously broken to $N = 1$ supergravity, with the survival of unbroken rather arbitrary gauge group. In order to open new possibilities for phenomenological model building we still have to face the problem of breaking the symmetry between the fermions and their *mirror* fields. It is also an interesting open question to find the relation of our mechanism with the non-perturbative $N = 2$ supergravities predicted by string–string duality and with the conjectured non perturbative breaking caused by extremal black-holes [37].

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